Generalized Measures of Fault Tolerance in Exchanged Hypercubes *

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Abstract

The exchanged hypercube $EH(s,t)$, proposed by Loh *et al.* [The exchanged hypercube, IEEE Transactions on Parallel and Distributed Systems 16 (9) (2005) 866-874], is obtained by removing edges from a hypercube Q_{s+t+1}. This paper considers a kind of generalized measures $\kappa^{(h)}$ and $\lambda^{(h)}$ of fault tolerance in $EH(s,t)$ with $1 \leqslant s \leqslant t$ and determines $\kappa^{(h)}(EH(s,t)) = \lambda^{(h)}(EH(s,t)) = 2^h(s+1-h)$ for any h with $0 \leqslant h \leqslant s$. The results show that at least $2^h(s+1-h)$ vertices (resp. $2^h(s+1-h)$ edges) of $EH(s,t)$ have to be removed to get a disconnected graph that contains no vertices of degree less than h, and generalizes some known results.

Keywords: Combinatorics, networks, fault-tolerant analysis, exchanged hypercube, connectivity, super connectivity

1 Introduction

It is well known that interconnection networks play an important role in parallel computing/communication systems. An interconnection network can be modeled by a graph $G = (V, E)$, where V is the set of processors and E is the set of communication links in the network. For graph terminology and notation not defined here we follow [15].

A subset $S \subset V(G)$ (resp. $F \subset E(G)$) of a connected graph G is called a *vertex-cut* (resp. *edge-cut*) if $G - S$ (resp. $G - F$) is disconnected. The *connectivity* $\kappa(G)$ (resp. *edge-connectivity* $\lambda(G)$) of G is defined as the minimum cardinality over all vertex-cuts (resp. edge-cuts) of G. The connectivity $\kappa(G)$ and edge-connectivity $\lambda(G)$ of a graph G are two important measurements for fault tolerance of the network since the larger $\kappa(G)$ or $\lambda(G)$ is, the more reliable the network is.

Because the connectivity has some shortcomings, Esfahanian [1] proposed the concept of restricted connectivity, Latifi *et al.* [3] generalized it to restricted h-connectivity

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which can measure fault tolerance of an interconnection network more accurately than the classical connectivity. The concepts stated here are slightly different from theirs.

A subset $S \subset V(G)$ (resp. $F \subset E(G)$) of a connected graph G, if any, is called an *h-vertex-cut* (resp. *edge-cut*), if $G - S$ (resp. $G - F$) is disconnected and has the minimum degree at least h. The *h-connectivity* (resp. *edge-connectivity*) of G, denoted by $\kappa^{(h)}(G)$ (resp. $\lambda^{(h)}(G)$), is defined as the minimum cardinality over all h-vertex-cuts (resp. h-edge-cut) of G. It is clear that, for $h \geqslant 1$, if $\kappa^{(h)}(G)$ and $\lambda^{(h)}(G)$) exists, then $\kappa^{(h-1)}(G) \leqslant \kappa^{(h)}(G)$ and $\lambda^{(h-1)}(G) \leqslant \lambda^{(h)}(G)$. For any graph G and any integer h, determining $\kappa^{(h)}(G)$ and $\lambda^{(h)}(G)$ is quite difficult. In fact, the existence of $\kappa^{(h)}(G)$ and $\lambda^{(h)}(G)$ is an open problem so far when $h \geqslant 1$. Only a little knowledge of results have been known on $\kappa^{(h)}$ and $\lambda^{(h)}$ for particular classes of graphs and small h's, such as [2,4,5,8,10–14,16,17,19,20].

It is widely known that the hypercube Q_n has been one of the most popular interconnection networks for parallel computer/communication system. Xu [14] determined $\lambda^{(h)}(Q_n) = 2^h(n-h)$ for $h \leqslant n-1$, and Oh *et al.* [11] and Wu *et al.* [13] independently determined $\kappa^{(h)}(Q_n) = 2^h(n-h)$ for $h \leqslant n-2$.

This paper is concerned about the exchanged hypercubes $EH(s,t)$, proposed by Loh *et al.* [7]. As a variant of the hypercube, $EH(s,t)$ is a graph obtained by removing edges from a hypercube Q_{s+t+1}. It not only keeps numerous desirable properties of the hypercube, but also reduced the interconnection complexity. Very recently, Ma *et al.* [10] have determined $\kappa^{(1)}(EH(s,t)) = \lambda^{(1)}(EH(s,t)) = 2s$. We, in this paper, will generalize this result by proving that $\kappa^{(h)}(EH(s,t)) = \lambda^{(h)}(EH(s,t)) = 2^h(s+1-h)$ for any h with $0 \leqslant h \leqslant s$.

The proof of this result is in Section 3. In Section 2, we recall the structure of $EH(s,t)$ and some lemmas used in our proofs.

2 Definitions and lemmas

For a given position integer n, let $I_n = \{1, 2, \ldots, n\}$. The sequence $x_n x_{n-1} \cdots x_1$ is said a binary string of length n if $x_r \in \{0,1\}$ for each $r \in I_n$. Let $x = x_n x_{n-1} \cdots x_1$ and $y = y_n y_{n-1} \cdots y_1$ be two distinct binary string of length n. *Hamming distance* between x and y, denoted by $H(x,y)$, is the number of r's for which $|x_r - y_r| = 1$ for $r \in I_n$.

For a binary string $u = u_n u_{n-1} \cdots u_1 u_0$ of length $n+1$, we call u_r the r-th bit of u for $r \in I_n$, and u_0 the last bit of u, denote sub-sequence $u_j u_{j-1} \cdots u_{i+1} u_i$ of u by $u[j:i]$, i.e., $u[j,i] = u_j u_{j-1} \cdots u_{i+1} u_i$. Let

$$V(s,t) = \{u_{s+t} \cdots u_{t+1} u_t \cdots u_1 u_0 |\ u_0, u_i \in \{0,1\}, i \in I_{s+t}\}.$$

Definition 2.1 *The exchanged hypercube is an undirected graph $EH(s,t) = (V,E)$, where $s \geqslant 1$ and $t \geqslant 1$ are integers. The set of vertices V is $V(s,t)$, and the set of edges E is composed of three disjoint types E_1, E_2 and E_3.*

$$\begin{aligned}
E_1 = \;& \{uv \in V \times V |\ u[s+t:1] = v[s+t:1], u_0 \neq v_0\},\\
E_2 = \;& \{uv \in V \times V |\ u[s+t:t+1] = v[s+t:t+1],\\
& H(u[t:1], v[t:1]) = 1, u_0 = v_0 = 1\},\\
E_3 = \;& \{uv \in V \times V |\ u[t:1] = v[t:1],\\
& H(u[s+t:t+1], v[s+t:t+1]) = 1, u_0 = v_0 = 0\}.
\end{aligned}$$

Now we give an alternative definition of $EH(s,t)$.

Definition 2.2 *An exchanged hypercube $EH(s,t)$ consists of the vertex-set $V(s,t)$ and the edge-set E, two vertex $u = u_{s+t}\cdots u_{t+1}u_t\cdots u_1u_0$ and $v = v_{s+t}\cdots v_{t+1}v_t\cdots v_1v_0$ linked by an edge, called r-dimensional edge, if and only if the following conditions are satisfied:*
a). u and v differ exactly in one bit on the r-th bit or on the last bit.
b). if $r \in I_t$, then $u_0 = v_0 = 1$,
c). if $r \in I_{s+t} - I_t$, then $u_0 = v_0 = 0$.

The exchanged hypercubes $EH(1,1)$ and $EH(1,2)$ are shown in Figure 1.

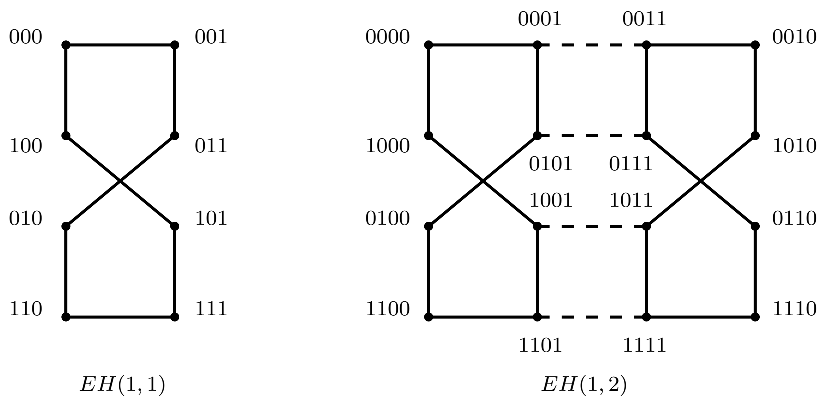

Figure 1: Two exchanged hypercubes $EH(1,1)$ and $EH(1,2)$

From Definition 2.2, it is easy to see that $EH(s,t)$ can be obtained from a hypercube Q_{s+t+1} with vertex-set $V(s,t)$ by removing all r-dimensional edges that link two vertices with the last bit 0 if $r \in I_t$ and two vertices with the last bit 1 if $r \in I_{s+t} - I_t$. Thus, $EH(s,t)$ is a bipartite graph with minimum degree $\min\{s,t\}+1$ and maximum degree $\max\{s,t\}+1$. The following three lemmas obtained by Loh *et al.* [7] and Ma [8] are very useful for our proofs.

Lemma 2.3 (Loh *et al.* [7]) *$EH(s,t)$ is isomorphic to $EH(t,s)$.*

By Lemma 2.3, without loss of generality, we can assume $s \leqslant t$ in the following discussion, and so $EH(s,t)$ has the minimum degree $s+1$. For fixed $r \in I_{s+t}$ and $i \in \{0,1\}$, let H_i^r denote a subgraph of $EH(s,t)$ induced by all vertices whose the r-th bits are i.

Lemma 2.4 (Loh *et al.* [7]) *For a fixed $r \in I_{s+t}$, $EH(s,t)$ can be decomposed into 2 isomorphic subgraphs H_0^r and H_1^r, which are isomorphic to $EH(s,t-1)$ if $r \in I_t$ and $t \geqslant 2$, and isomorphic to $EH(s-1,t)$ if $r \in I_{s+t} - I_t$ and $s \geqslant 2$. Moreover, there are 2^{s+t-1} independent edges between H_0^r and H_1^r.*

Lemma 2.5 (Ma [8]) $\kappa(EH(s,t)) = \lambda(EH(s,t)) = s+1$ *for any s and t with $1 \leqslant s \leqslant t$.*

3 Main results

In this section, we present our main results, that is, we determine the h-connectivity and h-edge-connectivity of the exchanged hypercube $EH(s,t)$.

Lemma 3.1 *$\kappa^{(h)}(EH(s,t)) \leqslant 2^h(s+1-h)$ and $\lambda^{(h)}(EH(s,t)) \leqslant 2^h(s+1-h)$ for $h \leqslant s$.*

Proof. Let X be a subset of vertices in $EH(s,t)$ whose the rightmost $s+t+1-h$ bits are zeros and the leftmost h bits do not care, denoted by

$$X = \{*^h 0^{s+t+1-h} |\ * \in \{0,1\}\}.$$

Then the subgraph of $EH(s,t)$ induced by X is a hypercube Q_h. Let S be the neighbor-set of X in $EH(s,t)-X$ and F the edge-sets between X and S. By Definition 2.2, S has the form

$$S = \{*^h \underbrace{0^p 1 0^{s-h-p-1}}_{s-h} 0^{t+1} |\ 0 \leqslant p \leqslant s-h-1,\ * \in \{0,1\}\} \cup \{*^h 0^{s+t-h} 1\}.$$

On the one hand, since every vertex of X has degree $s+1$ in $EH(s,t)$ and h neighbors in X, it has exactly $s-h+1$ neighbors in S. On the other hand, every vertex of S has exactly one neighbor in X. It follows that

$$|S| = |F| = 2^h(s+1-h).$$

We show that S is an h-vertex-cut of $EH(s,t)$. Clearly, S is a vertex-cut of $EH(s,t)$ since $|X \cup S| = 2^h(s+2-h) < 2^{s+t+1}$. Let $Y = EH(s,t) - (X \cup S)$ and v be any vertex in Y. We only need to show that the vertex v has degree at least h in Y. In fact, it is easy to see from the formal definition of S that if v is adjacent to some vertex in S then it has only the form

$$v = *^h \underbrace{0^p 1 0^{s-h-p-1}}_{s-h} 0^t 1 \quad \text{or } *^h\, 0^{s-h} \underbrace{0^r 1 0^{t-r-1}}_{t} 1 \text{ or } *^h \underbrace{0^p 1 0^q 1 0^{s-h-p-q-2}}_{s-h} 0^{t+1}$$

If v has the former two forms, then v has one neighbor in S, thus v has at least $(s+1-1 = s \geqslant)\, h$ neighbors in Y. If v has the last form, then $s-h \geqslant 2$ and v has two neighbors in S. Thus, v has at least $(s+1-2 = s-1 >)\, h$ neighbors in Y.

By the arbitrariness of $v \in Y$, S is an h-vertex-cut of $EH(s,t)$, and so

$$\kappa^{(h)}(EH(s,t)) \leqslant |S| = 2^h(s+1-h)$$

as required.

We now show that F is an h-edge-cut of $EH(s,t)$. Since every vertex v in $EH(s,t)-X$ has at most one neighbor in X, then v has at least $(s+1-1 = s \geqslant)\, h$ neighbors in $EH(s,t)-X$. By the arbitrariness of $v \in EH(s,t)-X$, F is an h-edge-cut of $EH(s,t)$, and so

$$\lambda^{(h)}(EH(s,t)) \leqslant |F| = 2^h(s+1-h)$$

The lemma follows. ∎

Corollary 3.2 *$\kappa^{(1)}(EH(1,t))=\lambda^{(1)}(EH(1,t))=2$ for $t\geqslant 1$.*

Proof. On the one hand, $\kappa^{(h)}(EH(1,t))\leqslant 2$ and $\lambda^{(h)}(EH(1,t))\leqslant 2$ by Lemma 3.1 when $s=1$. On the other hand, by Lemma 2.5, $\kappa(EH(1,t))=\lambda(EH(1,t))=2$, thus $\kappa^{(h)}(EH(1,t))\geqslant\kappa(EH(1,t))=2$ and $\lambda^{(h)}(EH(1,t))\geqslant\lambda(EH(1,t))=2$. The results hold. ∎

Theorem 3.3 *For $1\leqslant s\leqslant t$ and any h with $0\leqslant h\leqslant s$,*

$$\kappa^{(h)}(EH(s,t))=\lambda^{(h)}(EH(s,t))=2^h(s+1-h).$$

Proof. By Lemma 3.1, we only need to prove that,

$$\kappa^{(h)}(EH(s,t))=\lambda^{(h)}(EH(s,t))\geqslant 2^h(s+1-h).$$

We proceed by induction on $h\geqslant 0$. The theorem holds for $h=0$ by Lemma 2.5. Assume the induction hypothesis for $h-1$ with $h\geqslant 1$, that is,

$$\kappa^{(h-1)}(EH(s,t))=\lambda^{(h-1)}(EH(s,t))\geqslant 2^{h-1}(s+2-h). \tag{3.1}$$

Note $h=1$ if $s=1$. By Corollary 3.2, $\kappa^{(1)}(EH(1,t))=\lambda^{(1)}(EH(1,t))=2$ for any $t\geqslant 1$, the theorem is true for $s=1$. Thus, we assume $s\geqslant 2$ below.

Let S be a minimum h-vertex-cut (or h-edge-cut) of $EH(s,t)$ and X be the vertex-set of a minimum connected component of $EH(s,t)-S$. Then

$$|S|=\begin{cases}\kappa^{(h)}(EH(s,t)) & \textit{if } S \textit{ is a vertex}-\textit{cut};\\ \lambda^{(h)}(EH(s,t)) & \textit{if } S \textit{ is an edge}-\textit{cut}.\end{cases}$$

Thus, we only need to prove that

$$|S|\geqslant 2^h(s+1-h). \tag{3.2}$$

To the end, let Y be the set of vertices in $EH(s,t)-S$ not in X, and for a fixed $r\in I_{s+t}$ and each $i=0,1$, let

$$\begin{aligned}&X_i=X\cap H_i^r,\\ &Y_i=Y\cap H_i^r \text{ and}\\ &S_i=S\cap H_i^r,\end{aligned}$$

Let $J=\{i\in\{0,1\}|\ X_i\neq\emptyset\}$ and $J'=\{i\in J|\ Y_i\neq\emptyset\}$. Clearly, $0\leqslant|J'|\leqslant|J|\leqslant 2$ and $|J'|=0$ only when $|J|=1$. We choose $r\in I_{s+t}$ such that $|J|$ is as large as possible. For each $i\in\{0,1\}$, we write H_i for H_i^r for short. We first prove the following inequality.

$$|S_i|\geqslant 2^{h-1}(s+1-h) \text{ if } X_i\neq\emptyset \text{ and } Y_i\neq\emptyset \text{ for } i\in\{0,1\}. \tag{3.3}$$

In fact, for some $i\in\{0,1\}$, if $X_i\neq\emptyset$ and $Y_i\neq\emptyset$, then S_i is a vertex-cut (or an edge-cut) of H_i. Let u be any vertex in $X_i\cup Y_i$. Since S is an h-vertex-cut (or h-edge-cut) of $EH(s,t)$, u has degree at least h in $EH(s,t)-S$. By Lemma 2.4, u has at most one neighbor in H_j, where $j\neq i$. Thus, u has degree at least $h-1$ in H_i, which implies that S_i is an $(h-1)$-vertex-cut (or edge-cut) of H_i, that is,

$$|S_i|\geqslant\kappa^{(h-1)}(H_i)\ \ (\text{or } |S_i|\geqslant\lambda^{(h-1)}(H_i)). \tag{3.4}$$

If $r \in I_{s+t} - I_t$, then $H_i \cong EH(s-1,t)$ by Lemma 2.4. By the induction hypothesis (3.1), $\kappa^{(h-1)}(H_i) = \lambda^{(h-1)}(H_i) \geqslant 2^{h-1}(s+1-h)$, from which and (3.4), we have that $|S_i| \geqslant 2^{h-1}(s+1-h)$.

If $r \in I_t$, then $H_i \cong EH(s,t-1)$ by Lemma 2.4.

If $t \geqslant s+1$, by the induction hypothesis (3.1),

$$\kappa^{(h-1)}(H_i) = \lambda^{(h-1)}(H_i) \geqslant 2^{h-1}(s+2-h) > 2^{h-1}(s+1-h),$$

from which and (3.4), we have that $|S_i| > 2^{h-1}(s+1-h)$.

If $t = s$, then $EH(s,t-1) \cong EH(s-1,t)$ by Lemma 2.3. By the induction hypothesis (3.1),

$$\kappa^{(h-1)}(H_i) = \lambda^{(h-1)}(H_i) \geqslant 2^{h-1}(s+1-h),$$

from which and (3.4), we have that $|S_i| \geqslant 2^{h-1}(s+1-h)$. The inequality (3.3) follows.

We now prove the inequality in (3.2).

If $|J| = 1$ then, by the choice of J, no matter what $r \in I_{s+t}$ is chosen, the r-th bits of all vertices in X are the same. In other words, the r-th bits of all vertices in X are the same for any $r \in I_{s+t}$, and possible different in the last bit. Thus $|X| \leqslant 2$ and $h \leqslant 1$. By the hypothesis of $h \geqslant 1$, we have $h = 1$ and $|X| = 2$. The subgraph of $EH(s,t)$ induced by X is an edge in E_1, thus

$$|S| = s+t \geqslant 2s = 2^h(s+1-h),$$

as required. Assume $|J| = 2$ below, that is, $X_i \neq \emptyset$ for each $i = 0,1$. In this case, $|J'| \geqslant 1$.

If $|J'| = 2$ then, for each $i = 0,1$, since $X_i \neq \emptyset$ and $Y_i \neq \emptyset$, we have that $|S_i| \geqslant 2^{h-1}(s+1-h)$ by (3.3). Note that $|S| = |S_0| + |S_1|$ if S is an h-vertex-cut and $|S| \geqslant |S_0| + |S_1|$ if S is an h-edge-cut. It follows that

$$\begin{aligned} |S| &\geqslant |S_0| + |S_1| \\ &\geqslant 2 \times 2^{h-1}(s+1-h) \\ &= 2^h(s+1-h), \end{aligned}$$

as required.

If $|J'| = 1$, then one of Y_0 and Y_1 must be empty. Without loss of generality, assume $Y_1 = \emptyset$ and $Y_0 \neq \emptyset$.

Clearly, S is not an h-edge-cut, otherwise, $|Y| < |H_0| < |X|$, a contradiction with the minimality of X. Thus, S is an h-vertex-cut. By (3.3), $|S_0| \geqslant 2^{h-1}(s+1-h)$. Since $Y_1 = \emptyset$, we have

$$|X_1| = |H_1| - |S_1| \text{ and } |Y| = |H_0| - |X_0| - |S_0|. \tag{3.5}$$

If $|S_1| < |S_0|$ then, by (3.5), we obtain that $|Y| < |X_1| < |X|$, which contradicts to the minimality of X. Thus, $|S_1| \geqslant |S_0|$, from which and (3.3) we have that

$$\begin{aligned} |S| &= |S_0| + |S_1| \geqslant 2|S_0| \\ &\geqslant 2 \times 2^{h-1}(s+1-h) \\ &= 2^h(s+1-h), \end{aligned}$$

as required. Thus, the inequality in (3.2) holds, and so the theorem follows. ∎

Corollary 3.4 (Ma and Zhu [10]) *If* $1 \leqslant s \leqslant t$, *then* $\kappa^{(1)}(EH(s,t)) = \lambda^{(1)}(EH(s,t)) = 2s$.

A dual-cube $DC(n)$, proposed by Li and Peng [6] constructed from hypercubes, preserves the main desired properties of the hypercube. Very recently, Yang and Zhou [18] have determined that $\kappa^{(h)}(DC(n)) = 2^n(n+1-h)$ for each $h = 0,1,2$. Since $EH(n,n)$ is isomorphic to $DC(n)$, the following result is obtained immediately.

Corollary 3.5 *For dual-cube* $DC(n)$, $\kappa^{(h)}(DC(n)) = \lambda^{(h)}(DC(n)) = 2^n(n+1-h)$ *for any* h *with* $0 \leqslant h \leqslant n$.

4 Conclusions

In this paper, we consider the generalized measures of of fault tolerance for a network, called the h-connectivity κ^h and the h-edge-connectivity λ^h. For the exchanged hypercube $EH(s,t)$, which has about half edges of the hypercube Q_{s+t+1}, we prove that $\kappa^{(h)} = \lambda^{(h)} = 2^h(s+1-h)$ for any h with $0 \leqslant h \leqslant s$ and $s \leqslant t$. The results show that at least $2^h(s+1-h)$ vertices (resp. $2^h(s+1-h)$ edges) of $EH(s,t)$ have to be removed to get a disconnected graph that contains no vertices of degree less than h. Thus, when the exchanged hypercube is used to model the topological structure of a large-scale parallel processing system, these results can provide more accurate measurements for fault tolerance of the system.

Otherwise, Ma and Liu [9] investigated bipancyclicity of $EH(s,t)$. However, there are many interesting combinatorial and topological problems, e.g., wide-diameter, fault-diameter, panconnectivity, spanning-connectivity, which are still open for the exchanged hypercube network.

References

[1] A. H. Esfahanian, Generalized measures of fault tolerance with application to n-cube networks. IEEE Transactions on Computers, 38 (11) (1989), 1586-1591.

[2] A. H. Esfahanian, S.L. Hakimi, On computing a conditional edge connectivity of a graph. Information Processing Letters, 27 (1988),195-199.

[3] S. Latifi, M. Hegde, M. Naraghi-Pour, Conditional connectivity measures for large multiprocessor systems. IEEE Transactions on Computers, 43 (1994) 218-222.

[4] X.-J. Li and J.-M. Xu, Generalized measures of fault tolerance in (n,k)-star graphs. http://arxiv.org/abs/1204.1440, 2012.

[5] X.-J. Li and J.-M. Xu, Generalized measures of edge fault tolerance in (n,k)-star graphs. Mathematical Science Letters, 1 (2) (2012), 133-138.

[6] Y. Li, S. Peng, Dual-cubes: a new interconnection network for high-performance computer clusters. In: Proceedings of the 2000 International Computer Architecture, (2000), pp. 51-57.

[7] P. K. K. Loh, W. J. Hsu, Y. Pan, The exchanged hypercube. IEEE Transactions on Parallel and Distributed Systems, 16 (9) (2005), 866-874.

[8] M. Ma, The connectivity of exchanged hypercubes. Discrete Mathematics Algorithms and Applications, 2 (2) (2010), 213-220.

[9] M. Ma, B. Liu, Cycles embedding in exchanged hypercubes. Information Processing Letters, 110 (2) (2009), 71-76.

[10] M. Ma and L. Zhu, The super connectivity of exchanged hypercubes. Information Processing letters, 111 (2011), 360-364.

[11] A. D. Oh, H. Choi, Generalized measures of fault tolerance in n-cube networks. IEEE Transactions on Parallel and Distributed Systems, 4 (1993), 702-703.

[12] M. Wan, Z. Zhang, A kind of conditional vertex connectivity of star graphs. Applied Mathematics Letters, 22 (2009), 264-267.

[13] J. Wu, G. Guo, Fault tolerance measures for m-ary n-dimensional hypercubes based on forbidden faulty sets. IEEE Transactions on Computers, 47 (1998), 888-893.

[14] J.-M. Xu, On conditional edge-connectivity of graphs. Acta Mathematae Applicatae Sinica, 16 (4) (2000), 414-419.

[15] J.-M. Xu, Topological Structure and Analysis of Interconnection Networks. Kluwer Academic Publishers, Dordrecht/Boston/London, 2001.

[16] J.-M. Xu, M. Xu, Q. Zhu, The super connectivity of shuffle-cubes. Information Processing Letters, 96 (2005), 123-127.

[17] W.-H. Yang, H.-Z. Li, X.-F, Guo, A kind of conditional fault tolerance of (n, k)-star graghs. Information Processing Letters, 110 (2010), 1007-1011.

[18] X. Yang, S. Zhou, On conditional fault tolerant of dual-cubes. International Journal of Parallel, Emergent and Distributed Systems. DOI:10.1080/17445760.2012.704631, 2012

[19] Q. Zhu, J.-M. Xu, X.-M. Hou, X. Xu, On reliability of the folded hypercubes, Information Sciences, 177 (8) (2007), 1782-1788.

[20] Q. Zhu, J.-M. Xu, M. Lü, Edge fault tolerance analysis of a class of interconnection networks. Applied Mathematics and Computation, 172 (1) (2006), 111-121.